Inversion of Generalized Parabolic Projections[1]

Ali Özbek and Bernard C. Levy

Department of Electrical Engineering and Computer Science
and
Laboratory of Information and Decision Systems
Massachusetts Institute of Technology, Cambridge, MA 01239

Abstract

The multidimensional inverse scattering problem for an acoustic medium is considered within the homogeneous background Born approximation. A constant density acoustic medium is probed by a wide-band plane wave source, and the scattered field is observed along a receiver array located outside the medium. The inversion problem is formulated as a generalized tomographic problem. It is shown that the observed scattered field can be appropriately filtered so as to obtain generalized projections of the scattering potential. For a 2-D experimental geometry, these projections are weighted integrals of the scattering potential over regions of parabolic support. The inversion problem is therefor similar to that of x-ray tomography, except that instead of being given projections of the object to be reconstructed along straight lines, projections along parabolas are given. The inversion procedure that we propose is similar to the x-ray solution, in the sense that it consists of a backprojection operation followed by 2-D space invariant filtering. A "Projection-Slice Theorem" is derived relating the generalized projections and the scattering potential in the Fourier transform domain.

1. Introduction

In inverse scattering problems, the objective is to reconstruct certain physical properties of a medium from scattering experiments. In general, there is an array of sources and an array of receivers located outside the medium. There are three general approaches to the inversion problem: Iterative inversion, exact direct inversion and approximate direct inversion. The iterative inversion (also called generalized inversion) approach attempts to solve a very large scale least-squares problem, where at every stage an estimate of the medium parameters is used to solve the forward scattering problem for the corresponding wave field at the receiver locations. Depending on the difference between the observed and the computed scattered waves, a new estimate of the medium model is obtained and the next iteration is performed. These methods are very time consuming, and the convergence of the current methods depends on the accuracy of the initial estimate.

The objective of exact direct inversion methods is to reconstruct the medium properties exactly, with no iterations involved. These methods require large numbers of sources and receivers with particular observation geometries which limit their applicability to practical problems. From a theoretical point of view, one-dimensional exact direct inverse scattering methods have reached an advanced level of development (see [1] for an overview), whereas their extension to higher dimensions has proved to be difficult.

Consequently, the logical approach to the practical solution of the multidimensional inverse scattering problems is the use of approximate direct inversion methods. The differential equation for wave propagation in a medium can be transformed into the so-called Lippmann-Schwinger integral equation [12]. For example, for an acoustic medium with constant density, this equation expresses in integral form the scattered field inside the medium in terms of the propagation velocity profile and the total pressure field inside the medium. The Born approximation consists of approximating the total field inside the integral representation by the incident field. Therefore, this approximation assumes that the scattered field is small compared to the incident field; or equivalently that the perturbations of the velocities are small with respect to the background velocity profile which is used to compute the incident field. Another way of interpreting the Born approximation is to view it as a linearization technique, where the relation between the scattered field and the object profile that we want to reconstruct is linear. Physically, the Born approximation takes into account only the singly scattered waves; multiply scattered waves are considered as noise. Note that, depending on the

[1] This work was supported by the Air Force Office of Scientific Research under Grant No. AFOSR-85-0227 and by the National Science Foundation under Grant No. ECS-83-12921.

Report Documentation Page

Form Approved
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE **MAR 1987**	2. REPORT TYPE	3. DATES COVERED **00-03-1987 to 00-03-1987**

4. TITLE AND SUBTITLE **Inversion of Generalized Parabolic Projections**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Massachusetts Institute of Technology,Laboratory for Information and Decision Systems,77 Massachusetts Avenue,Cambridge,MA,02139-4307**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**		**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

method used to compute the background field, the multiples due to the background model may be included in the scattered field. The multiples due to the residual velocities are neglected.

In this paper, the inverse scattering problem for an acoustic medium is considered within the homogeneous background Born approximation. A constant density acoustic medium is probed by a wide-band plane wave source, and the scattered field is observed along a receiver array located outside the medium. The objective is to reconstruct the scattering potential, which is a function of the propagation velocity inhomogeneities in the medium. The monochromatic plane-wave source inverse scattering problem has been studied under the name of diffraction tomography by several people, including Mueller, Kaveh, Devaney and Beylkin [3,4,7]. Roberts and Kak investigated the reflection mode problem with broadband illumination [11], whereas Esmersoy and Levy presented a solution in terms of an extrapolated field [5].

In the present paper, the key observation is that, the time traces observed at the receivers can be appropriately filtered so as to obtain generalized projections of the scattering potential. For a two-dimensional experimental geometry, these projections are weighted integrals of the scattering potential over regions of parabolic support, whereas they become surface integrals over paraboloids for the three-dimensional case. Thus the inverse scattering problem is now posed as a generalized tomographic or integral geometric problem.

The straight-line tomography problem, which arises in x-ray tomography, was first solved by Radon [10]; see Deans [2] for a full treatment. Fawcett [6] formulated the zero-offset Born inversion problem as a generalized tomographic problem, where the objective is to reconstruct a function from its projections along circles or spheres.

The reconstruction problem for parabolical projections can be formulated in a way similar to the problem of x-ray tomography. The solution can be expressed as a backprojection operation where we sum the contributions of all projections passing through a given point in space, followed by a two or three dimensional filtering operation.

A different, physically-oriented interpretation of the backprojection operation appearing in our reconstruction technique can be developed by showing that it corresponds to first backpropagating the observed filtered time traces, and then correlating the backpropagated field with the incident probing wave field. The details of this interpretation will be reported in [8]. A "Projection-Slice Theorem" is also derived relating the generalized projections and the scattering potential in the Fourier transform domain.

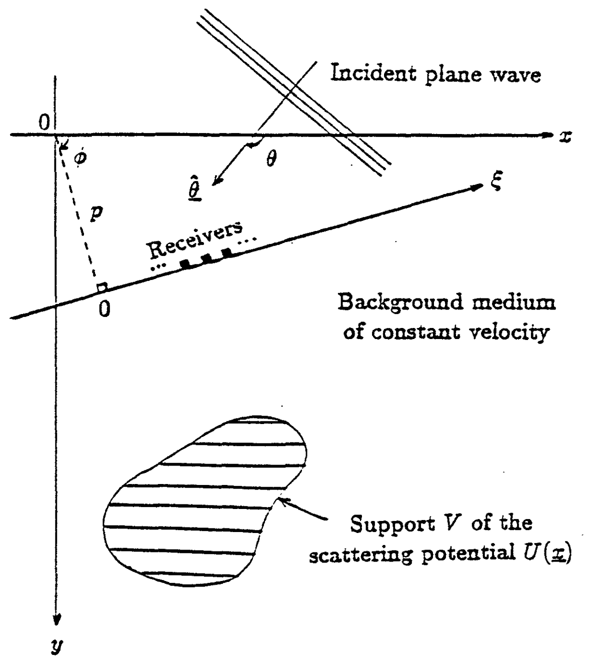

Fig. 1 The experimental geometry.

2. Problem Description

In this paper we will treat the two-dimensional case; a more detailed account including the three-dimensional case will be reported elsewhere [8]. Consider the scattering experiment described in Fig. 1. A constant density 2-D acoustic medium is probed by a wide-band plane wave and the scattered field is observed along a straight-line receiver array. The Fourier transform $P(\underline{x},\omega)$ of the pressure field at position $\underline{x} = (x, y)$ satisfies

$$[\nabla^2 + k^2 n^2(\underline{x})]P(\underline{x},\omega) = 0, \tag{1}$$

where $k = \omega/c_0$ is the wavenumber, $n(\underline{x}) \doteq c_0/c(\underline{x})$ is the refractive index of the medium, $c(\underline{x})$ is the propagation velocity at point $\underline{x}$ and c_0 is the propagation velocity of the background medium. We assume that $n(\underline{x})$ does not deviate significantly from the background index 1, so that

$$n^2(\underline{x}) = 1 + U(\underline{x}), \tag{2}$$

where the scattering potential $U(\underline{x})$ is small. We also assume that $U(\underline{x})$ has a bounded support V, which is completely located on the same side of the receiver array, as shown in Fig. 1. Decomposing the total field P into the incident field P_0 and the scattered field

$$P(\underline{x},\omega) \equiv P_0(\underline{x},\omega) + P_s(\underline{x},\omega), \tag{3}$$

and noting that the incident field P_0 satisfies the Helmholtz equation associated with the background medium

$$(\nabla^2 + k^2)P_0(\underline{x}, \omega) = 0, \tag{4}$$

we find that the scattered field P_s satisfies the equation

$$(\nabla^2 + k^2)P_s(\underline{x}, \omega) = -k^2 U(\underline{x}) P(\underline{x}, \omega). \tag{5}$$

The solution of (5) is given by the Lippmann-Schwinger equation [12]

$$P_s(\underline{x}, \omega) = k^2 \int d\underline{x}' U(\underline{x}') G_0(\underline{x}, \underline{x}', \omega) P(\underline{x}', \omega), \tag{6}$$

where $G_0(\underline{x}, \underline{x}', \omega)$ is the Green's function associated with a point source in a homogeneous medium:

$$(\nabla^2 + k^2) G_0(\underline{x}, \underline{x}', \omega) = -\delta(\underline{x} - \underline{x}'). \tag{7}$$

Equation (6) demonstrates the nonlinear relation that exists between the potential $U(\underline{x})$ and the pressure field $P(\underline{x}, \omega)$. To linearize this equation we adopt the Born approximation, whereby we assume $P_s(\underline{x}, \omega) \ll P_0(\underline{x}, \omega)$. Hence the Lippmann-Schwinger equation becomes

$$P_s(\underline{x}, \omega) \approx k^2 \int d\underline{x}' U(\underline{x}') G_0(\underline{x}, \underline{x}', \omega) P_0(\underline{x}', \omega). \tag{8}$$

For the problem under consideration, the Green's function and the incident wave are given as

$$G_0(\underline{x}, \underline{x}', \omega) = \frac{i}{4} H_0^{(1)}(k|\underline{x} - \underline{x}'|), \tag{9}$$

$$P_0(\underline{x}', \omega) = e^{ik\hat{\underline{\theta}}\cdot\underline{x}'}, \tag{10}$$

where $\hat{\underline{\theta}}$ is the unit vector which indicates the angle of incidence of the plane-wave source, and $H_0^{(1)}(\cdot)$ indicates the Hankel function of order zero and type one. Therefore, the scattered field at a receiver point

$$\underline{\xi} = (p\cos\phi + \xi\sin\phi,\ p\sin\phi - \xi\cos\phi)$$

is given by

$$P_s(\underline{\xi}, \omega) = \frac{ik^2}{4} \int d\underline{x}' U(\underline{x}') e^{ik\hat{\underline{\theta}}\cdot\underline{x}'} H_0^{(1)}(k|\underline{x}' - \underline{\xi}|), \tag{11}$$

within the Born approximation. Hence, we have

$$\begin{aligned} \frac{2\pi}{k^2} P_s(\underline{\xi}, \omega) &= \frac{i\pi}{2} \int d\underline{x}' U(\underline{x}') e^{ik\hat{\underline{\theta}}\cdot\underline{x}'} H_0^{(1)}(k|\underline{x}' - \underline{\xi}|) \\ &\equiv \hat{F}(\xi, k). \end{aligned} \tag{12}$$

Define the inverse Fourier transform of $\hat{F}(\xi, k)$ with respect to k as $g(\xi, r)$:

$$g(\xi, r) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \hat{F}(\xi, k) e^{-ikr}. \tag{13}$$

Taking account of the fact that (see [9], p. 731)

$$\mathcal{F}^{-1}\left\{\frac{i\pi}{2} H_0^{(1)}(ku)\right\} = \frac{1(r-u)}{\sqrt{r^2 - u^2}}, \tag{14}$$

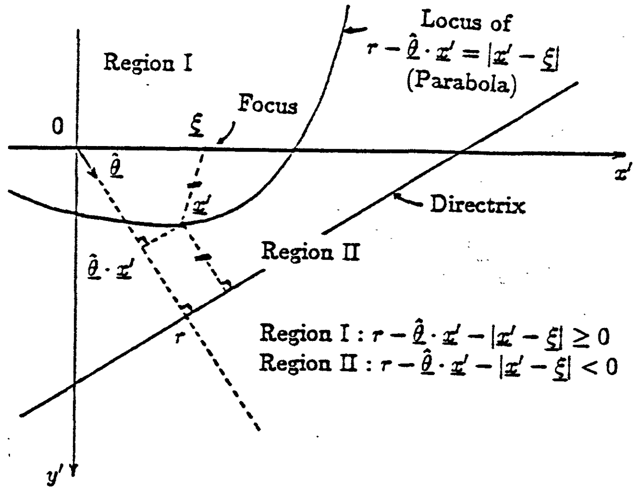

Fig. 2 The generalized parabolic projections.

where $1(\cdot)$ is the unit step function, therefore we find that

$$g(\xi, r) = \int d\underline{x}' U(\underline{x}') \frac{1(r - \hat{\underline{\theta}}\cdot\underline{x}' - |\underline{x}' - \underline{\xi}|)}{\sqrt{(r - \hat{\underline{\theta}}\cdot\underline{x}')^2 - |\underline{x}' - \underline{\xi}|^2}}. \tag{15}$$

This identity expresses $g(\xi, r)$ as a weighted integral of the scattering potential $U(\underline{x})$ where the weighting function is nonzero in a region with parabolic support. The parabola satisfies the equation $r = \hat{\underline{\theta}}\cdot\underline{x} - |\underline{x} - \underline{\xi}|$ where r, $\hat{\underline{\theta}}$ and $\underline{\xi}$ are given and $\underline{x}$ varies, as shown in Fig. 2. The weighting function becomes infinite for values of $\underline{x}$ along this parabola, so that the largest contribution to the integral is made by the values of $U(\underline{x})$ which lie along the parabola. In some sense, $g(\xi, r)$ is a projection of $U(\underline{x})$ with respect to a function whose singularities are algebraic and located along a parabola.

It is then interesting to note that the projections $g(\xi, r)$ can be obtained directly from the observed scattered field $P_s(\underline{\xi}, t)$ in time domain also: From (12) and (13),

$$g(\xi, r) = -2\pi c_0 \int_0^{r/c_0} d\tau \int_0^{\tau} ds P_s(\underline{\xi}, s). \tag{16}$$

Thus, the projections $g(\xi, r)$ are proportional to the scattered field twice integrated.

Therefore, given the observed scattered field $P_s(\xi, t)$ for ξ along the line array depicted in Fig. 2, and for $-\infty \leq t < \infty$, it is a straightforward procedure to find the generalized projection $g(\xi, r)$ for $-\infty < \xi < \infty$ and $0 \leq r < \infty$, and in the following it will be assumed that these projections constitute the data that is given by the scattering experiment. From this point of view,

the inverse scattering problem can be formulated as follows: Given the generalized projections

$$\{g(\xi, r): \ -\infty < \xi < \infty, \ 0 \leq r < \infty\},$$

we want to reconstruct the scattering potential $U(\underline{x})$.

In some sense, the above reconstruction problem is similar to the problem of x-ray tomography, where we are given the projections

$$g(r, \theta) = \int d\underline{x}' U(\underline{x}') \delta(r - \underline{\hat{\theta}} \cdot \underline{x}')$$

of $U(\underline{x})$ along straight lines. It is also analog to the inverse scattering problem considered by Fawcett [6], where it was shown that the so-called zero-offset inverse scattering geometry could be reduced to the solution of a generalized tomographic problem where the objective is to reconstruct a function $U(\underline{x})$ from its projections along circles of arbitrary radii centered along a straight line. There exists however an important difference between the problem of x-ray tomography, or the problem considered by Fawcett [6], and the problem that we examine here. In x-ray tomography, the projections of $U(\underline{x})$ are taken with respect to a weighting function which is the distribution $\delta(r - \underline{\hat{\theta}} \cdot \underline{\hat{x}})$, whereas the weighting function appearing in (15) is algebraic. Thus, in 2-D the generalized parabolic projection $g(\xi, r)$ is not an integral along a curve but an integral that has nonzero weighting over a whole region of the plane (the inside of the parabola depicted in Fig. 2). This is due to the fact that the 2-D Green's function does not have an impulsive waveform, but has a tail, as indicated by equation (9).

3. The Backprojection Operation

Like the x-ray tomography inversion procedure, or the method proposed by Fawcett [6] for the case of circular projections, the first step of our inversion procedure is to perform a backprojection operation on the projections $g(\xi, r)$. At a given point $\underline{x}$, this operation sums the contributions of the projections $g(\xi, r)$ which correspond to parabolic regions that include the point $\underline{x}$. In this summation, the projection $g(\xi, r)$ is weighted in proportion to the amount $U(\underline{x})$ has contributed to it according to the forward scattering equation (15). By performing this backprojection operation for every point in the plane, this gives a backprojection approximation, $U_B(\underline{x})$ to $U(\underline{x})$. It is given by

$$U_B(\underline{x}) \equiv \int_{-\infty}^{\infty} d\xi \int_0^{\infty} dr g(\xi, r) \frac{1(r - \underline{\hat{\theta}} \cdot \underline{x} - |\underline{x} - \underline{\xi}|)}{\sqrt{(r - \underline{\hat{\theta}} \cdot \underline{x})^2 - |\underline{x} - \underline{\xi}|^2}}. \tag{17}$$

Our first objective is to find a frequency domain relationship between $g(\xi, r)$ and $U_B(\underline{x})$. It can be shown that [8]

$$\int d\underline{x} e^{-i\underline{k}\cdot\underline{x}} \frac{1(r - \underline{\hat{\theta}} \cdot \underline{x} - |\underline{x} - \underline{\xi}|)}{\sqrt{(r - \underline{\hat{\theta}} \cdot \underline{x})^2 - |\underline{x} - \underline{\xi}|^2}} = \frac{i\pi}{k_x \cos\theta + k_y \sin\theta}$$

$$\cdot \exp\left\{-ip\frac{(k_x^2 - k_y^2)\cos(\theta + \phi) + 2k_x k_y \sin(\theta + \phi)}{2(k_x \cos\theta + k_y \sin\theta)}\right\}$$

$$\cdot \exp\left\{-i\xi\frac{(k_x^2 - k_y^2)\sin(\theta + \phi) - 2k_x k_y \cos(\theta + \phi)}{2(k_x \cos\theta + k_y \sin\theta)}\right\}$$

$$\cdot \exp\left\{-ir\frac{k^2}{2(k_x \cos\theta + k_y \sin\theta)}\right\}, \tag{18}$$

where $\underline{k} = (k_x, k_y)$ and $k = |\underline{k}|$. Therefore

$$\hat{U}_B(\underline{k}) \equiv \int d\underline{x} e^{-i\underline{k}\cdot\underline{x}} U_B(\underline{x}) = \frac{i\pi}{k_x \cos\theta + k_y \sin\theta}$$

$$\cdot \exp\left\{-ip\frac{(k_x^2 - k_y^2)\cos(\theta + \phi) + 2k_x k_y \sin(\theta + \phi)}{2(k_x \cos\theta + k_y \sin\theta)}\right\}$$

$$\cdot \hat{\hat{g}}\left(k_\xi = \frac{(k_x^2 - k_y^2)\sin(\theta + \phi) - 2k_x k_y \cos(\theta + \phi)}{2(k_x \cos\theta + k_y \sin\theta)},\right.$$

$$\left. k_r = \frac{k^2}{2(k_x \cos\theta + k_y \sin\theta)}\right), \tag{19}$$

where $\hat{\hat{g}}(k_\xi, k_r)$ is the 2-D Fourier transform of $g(\xi, r)$.

4. The Relationship Between $\hat{U}_B(\underline{k})$ and $\hat{U}(\underline{k})$

We first take the Fourier transform of $g(\xi, r)$ with respect to r:

$$\begin{aligned} \hat{g}(\xi, k_r) &= \int_0^{\infty} dr e^{-ik_r r} g(\xi, r) \qquad (20) \\ &= \hat{F}^*(\xi, k_r) \\ &= -\frac{i\pi}{2} \int d\underline{x}' U(\underline{x}') e^{ik\underline{\hat{\theta}}\cdot\underline{x}'} H_0^{(2)}(k|\underline{x}' - \underline{\xi}|), \end{aligned}$$

from (12). Now taking the Fourier transform with respect to ξ, we obtain [9]

$$\hat{\hat{g}}(k_\xi, k_r) = -\frac{i\pi \operatorname{sgn}(k_r)}{\sqrt{k_r^2 - k_\xi^2}} e^{i\gamma p \operatorname{sgn}(k_r)\sqrt{k_r^2 - k_\xi^2}}$$

$$\cdot \hat{U}(k_x = k_r \cos\theta + k_\xi \sin\phi + \gamma \operatorname{sgn}(k_r)\sqrt{k_r^2 - k_\xi^2}\cos\phi,$$

$$k_y = k_r \sin\theta - k_\xi \cos\phi + \gamma \operatorname{sgn}(k_r)\sqrt{k_r^2 - k_\xi^2}\sin\phi),$$

$$\text{for } |k_\xi| < |k_r|,$$

$$\hat{\hat{g}}(k_\xi, k_r) = 0, \quad \text{for } |k_\xi| \geq |k_r|, \tag{21}$$

and where

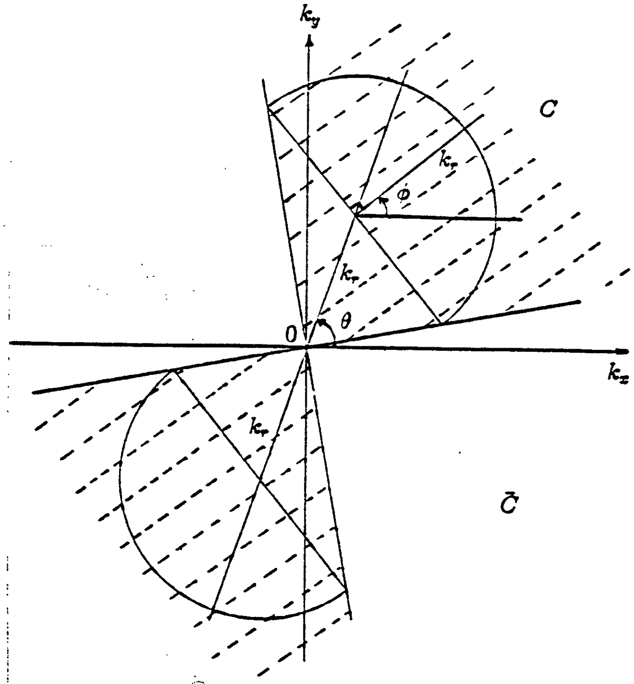

Fig. 3 The coverage of $\hat{U}(\underline{k})$.

$$\gamma = \begin{cases} +1 & \text{if } (x\cos\phi + y\sin\phi - p) \geq 0 \text{ for all } x, y \in V, \\ -1 & \text{if } (x\cos\phi + y\sin\phi - p) < 0 \text{ for all } x, y \in V. \end{cases}$$

For normal incidence ($\theta = \phi$) and fixed k_r, this result corresponds to the formula obtained by Devaney in the context of diffraction tomography [3]. This relationship is similar to the Projection Slice Theorem of straight-line tomography (see [2], Section 6.2). It relates the 1-D Fourier transform of $\hat{g}(\xi, k_r)$ with respect to ξ, and the 2-D Fourier transform of $U(\underline{x})$. For a fixed k_r, $\hat{\hat{g}}(k_\xi, k_r)$ gives $\hat{U}(\underline{k})$ along two semicircles of radius k_r centered at $(k_r\cos\theta, k_r\sin\theta)$ and $(-k_r\cos\theta, -k_r\sin\theta)$ (see Fig. 3). By letting k_r vary, we see that $\hat{U}(\underline{k})$ is only known in the cone C, which is defined as

$$C = \left\{k_x, k_y : \frac{1}{2}\left(\theta + \phi - \frac{\pi}{2}\right) \leq \tan^{-1}\frac{k_y}{k_x} \leq \frac{1}{2}\left(\theta + \phi + \frac{\pi}{2}\right) \text{ or } \frac{1}{2}\left(\theta + \phi + \frac{3\pi}{2}\right) \leq \tan^{-1}\frac{k_y}{k_x} \leq \frac{1}{2}\left(\theta + \phi + \frac{5\pi}{2}\right)\right\} \tag{22}$$

for $\gamma = +1$. For $\gamma = -1$, C is the complement of the above.

The inverse formula of (21) is

$$\hat{U}(\underline{k}) = \frac{i[(k_x^2 - k_y^2)\cos(\theta+\phi) + 2k_xk_y\sin(\theta+\phi)]}{2\pi(k_x\cos\theta + k_y\sin\theta)} \cdot \exp\left\{-ip\frac{(k_x^2 - k_y^2)\cos(\theta+\phi) + 2k_xk_y\sin(\theta+\phi)}{2(k_x\cos\theta + k_y\sin\theta)}\right\} \cdot \hat{\hat{g}}\left(k_\xi = \frac{(k_x^2 - k_y^2)\sin(\theta+\phi) - 2k_xk_y\cos(\theta+\phi)}{2(k_x\cos\theta + k_y\sin\theta)}, k_r = \frac{k^2}{2(k_x\cos\theta + k_y\sin\theta)}\right), \underline{k} \in C. \tag{23}$$

Combining (19) and (23) gives

$$\hat{U}(\underline{k}) = \frac{1}{2\pi^2}\left[(k_x^2 - k_y^2)\cos(\theta+\phi) + 2k_xk_y\sin(\theta+\phi)\right]\hat{U}_B(\underline{k}), \tag{24}$$

for $\underline{k} \in C$. Therefore, $\hat{U}(\underline{k})$ can be obtained from $\hat{U}_B(\underline{k})$ by a single 2-D filtering operation. This is similar to the "filter of backprojections" method used in straight-line tomography (see [2], Section 6.5).

By using the generalized parabolical projections $g(\xi, r)$ obtained from a single straight-line array, we obtain the coverage of $\hat{U}(\underline{k})$ over a 90^0 cone. To obtain a more complete coverage, we can use additional receiver arrays or perform several experiments with plane-waves incident from several directions.

5. Numerical Example

We present computer simulation results for simulation. Fig. 4 shows the scattering potential to be reconstructed. It represents a ±5% variation in the propagation velocity with respect to the constant background velocity. In the experiments, the probing plane wave is incident perpendicularly to Side A and the receivers were placed on all four sides. The scattered waves were computed by a finite-difference algorithm. The source wavelet used was a Blackman-Harris window and the diameter of the object was twice the dominant wavelength. Fig. 5 shows the reconstruction obtained using only the receivers on Side A, while Figs. 6 and 7 show the same for the cases where the receivers are only located on Sides B and C respectively. Fig. 8 depicts the reconstruction using all the receivers.

6. Conclusion

In this paper we have considered the direct velocity inversion problem for a constant-density acoustic medium probed by a single wide-band plane wave. The problem was posed as a generalized tomographic problem, where weighted integrals of the scattering potential $U(\underline{x})$ over parabolic regions are considered as data.

Drawing analogy to x-ray tomography, a backprojection operator, $U_B(\underline{x})$ was defined, which can be viewed as a "migration" approximation to $U(\underline{x})$. $U_B(\underline{x})$ was related to the parabolical projections $g(\xi, r)$ in the 2-D Fourier transform domain. The parabolical projections were also related to $U(\underline{x})$ in the frequency domain, thus deriving a "Projection Slice Theorem".

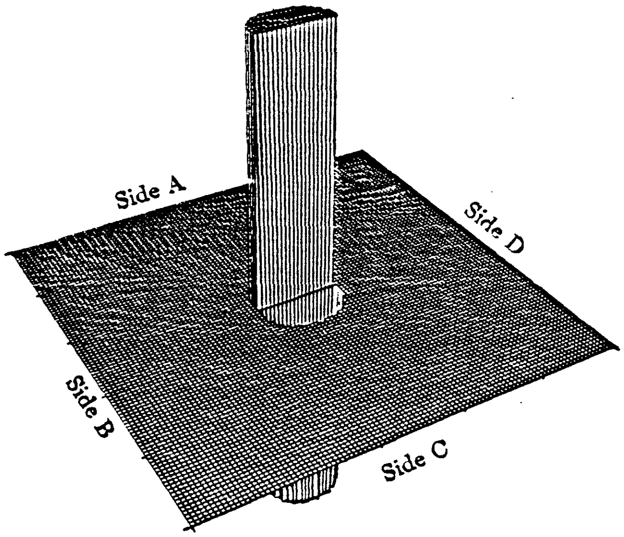

Fig. 4 The scattering potential for the synthetic example.

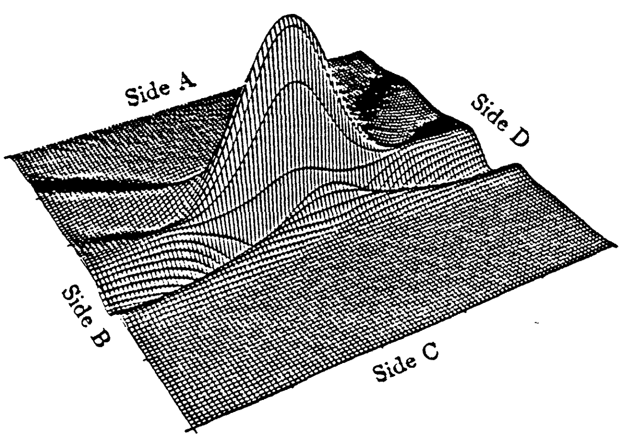

Fig. 5 Reconstruction using the receivers on Side A.

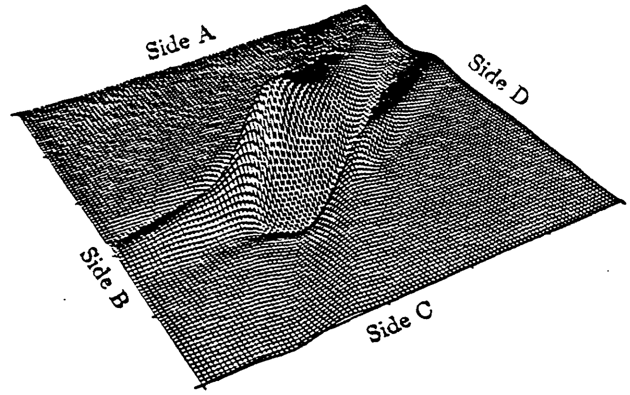

Fig. 6 Reconstruction using the receivers on Side B.

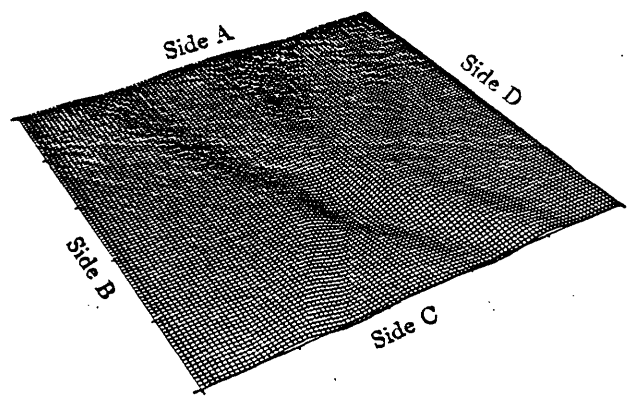

Fig. 7 Reconstruction using the receivers on Side C.

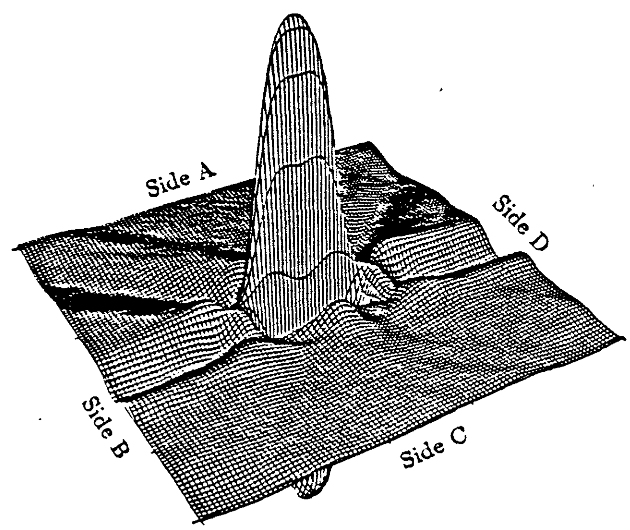

Fig. 8 Reconstruction using all the receivers.

Acknowledgements

The numerical example in this paper was prepared while the first author was a summer student at Schlumberger-Doll Research. He especially wishes to thank Dr. Cengiz Esmersoy and Dr. Douglas Miller for their assistance.

References

[1] A. M. Bruckstein, B. C. Levy, and T. Kailath, "Differential Methods in Inverse Scattering," *SIAM J. Appl. Math.*, Vol. 45, No. 2, pp. 312–335, April 1985.

[2] S. R. Deans, *The Radon Transform and Some of its Applications*, Wiley Interscience, New York, 1983.

[3] A. J. Devaney, "A Filtered Backpropagation Algorithm for Diffraction Tomography," *Ultrasonic Imaging* 4, pp. 336–350, 1982.

[4] A. J. Devaney and G. Beylkin, "Diffraction Tomography Using Arbitrary Transmitter and Receiver Surfaces," *Ultrasonic Imaging* 6, pp. 181–193, 1984.

[5] C. Esmersoy and B. C. Levy, "Multidimensional Born Inversion with a Wide-Band Plane-Wave Source," *Proc. IEEE*, Vol. 74, No. 3, pp. 466–475, March 1986.

[6] J. A. Fawcett, "Inversion of N-Dimensional Spherical Averages," *SIAM J. Appl. Math.*, Vol. 45, No. 2, pp. 336–341, April 1985.

[7] R. K. Mueller, M. Kaveh, and R. D. Inverson, "A New Approach to Acoustic Tomography Using Diffraction Techniques," in *Acoustic Imaging Vol. 8*, A. F. Metherell, ed., pp. 615–628, Plenum Press, New York, 1980.

[8] A. Özbek and B. C. Levy, "Inversion of Parabolic and Paraboloidal Projections," to be submitted to *IEEE Trans. on Acoust., Speech and Sign. Proc.*

[9] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, New York, 1980.

[10] J. Radon, "Über die Bestimmung von Funktionen durch ihre Integralwerte längs gewisser Mannigfaltigkeiten," *Berichte Sächsische Akademie der Wissenschaften. Leipzig, Math.-Phys. Kl.* 69, pp. 262–267, 1917.

[11] B. A. Roberts and A. C. Kak, "Reflection Mode Diffraction Tomography," *Ultrasonic Imaging* 7, pp. 300–320, 1985.

[12] J. R. Taylor, *Scattering Theory*, John Wiley, New York, 1972.